Gm vs mu

by Mike McCarty

Most tube testers just check emission. Some of them also check for some sort of gain; often a "transconductance" is measured. Most of them do *not* measure transconductance or Gm, but rather tube gain or mu. I've noticed some confusion here and there on the net, so I thought I'd explain the meanings of mu and Gm, and their relationship. Throughout this message, I use "voltage" as a synonym for "EMF".

A tube is modeled as a voltage-controlled current source. That is, at any given plate voltage, the current in the plate (and cathode) is proportional to the voltage on the grid. There are several things wrong with this model, as with any mathematical model. It's simplified, and not terribly accurate, but nonetheless it is very useful due in part to its simplicity.

Tubes are, of course, not linear, so exact proportionality is a fiction. But within bounds, this model gives a reasonable first approximation to what happens. A short enough piece of any smooth curve looks like a straight-line segment. It turns out that the curves associated with tubes are "straight enough" over about 2/3 of their reasonable operating conditions for straight-line approximations to be useful there.

So, what are the exact definitions? First, they do not involve DC or static conditions. These are for small value AC signals, or to put it another way, small modifications to the static bias conditions. Second, the exact mathematical definitions are in terms of things developed in calculus courses, called partial derivatives. If you haven't had calculus, don't worry. I'll give information, which doesn't involve it.

Gm is the ratio between the plate current change and the grid to cathode voltage change with plate to cathode voltage held constant. If we call Ip the plate current, and Vg the grid voltage, then

Gm = dIp / dVg

where you may read the "d" as "small change in". Another way to read it is as "The small AC plate current resulting from a small AC grid voltage, with the plate voltage held constant, or short circuited for AC" (like by bypassing the tube with a huge capacitor).

If you know calculus, then it means the partial derivative of plate current with respect to grid voltage. Its unit is that of current divided by voltage, or the mho or Siemens, and it is a sort of "conductance".

Since this "conductance" is from the input to the output circuits of the tube, and goes "across" the tube, it is called "transconductance" or "mutual conductance". Since "G" is the symbol used in electronics for conductance, this explains the use of the symbol "Gm". It is not a real conductance in any sense of the term.

In the fictional mathematical model, this is a single constant. With a real tube it depends on plate voltage, plate current, tube temperature, frequency of the signal, mood of the operator, etc.

Now mu is defined similarly, but it is a ratio of voltages

mu = dVp / dVg

This is the AC voltage gain for small signals, being the AC voltage in the plate divided by the AC voltage on the grid, with the plate current held constant. It has no unit, being a pure number. In the fictional model, this also is a single fixed constant for any given tube. For the mathematically inclined, it is the partial derivative of the plate voltage with respect to the grid voltage.

Now, also in the fictional model, there is an effective plate resistance associated with the tube, usually denoted by Rp. By definition, Rp = dVp / dIp, in other words the AC voltage on the plate divided by the AC current through the plate. Thus we have that

mu = Gm x Rp

That's why, in an earlier message, I said that Gm and mu are effectively the same. They are essentially proportional to each other. Well, not exactly of course. Gm is a conductance, and mu is a pure number. But in the fictional model Rp is *also* a constant (for any given tube, that is), so there you are.[1]

Now, mu is easier to measure, at least approximately, so that is what "transconductance testers" usually measure. A small AC voltage is placed on the grid of an otherwise appropriately biased tube, and the AC voltage on the plate is displayed, perhaps divided by the (fixed) AC grid voltage, giving an approximation to mu.

I have with me some data sheets for various tubes, which include the parameters for the fictional models, portions of which I reproduce here.

I give Gm in umhos, Rp in K ohms. The values of Gm, Rp, and mu are from the data sheets, while I compute Gm x Rp myself. These are all twin triodes, and the second element of the tube is indicated with an asterisk ("*") after it.

Tube	Gm	Rp	mu	Gm x Rp
12AX7	1250	80	100	100
12AX7*	1600	62.5	100	100
12AT7	4000	15	60	60
12AT7*	5500	10.9	60	59.95
12AU7A	3100	6.25	19.5	19.375
12AU7A*	2200	7.7	17	16.94

Actually, the data sheets simply divide mu by Gm to get Rp, and round off to a couple of figures.

[1] The mathematically adept will note that the Gm and mu definitions require different static conditions on the other circuit parameters, and hence it cannot be the case that mu = Gm x Rp. But this whole paper is "to first order".

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